INTRODUCTION

Far too often, minority students who enter college aspiring to careers in science do not survive introductory calculus, physics, and chemistry courses. These intellectually rigorous courses, which are the foundation for further work in technical fields, are difficult even for the well-prepared student unencumbered by the multiple adjustments to college life that minority students typically face. Yet traditional remedial and counseling approaches have rarely achieved significant results in assisting these students through the crucial first year of college, or, as important, in increasing the number of those who complete a professional math/science-based curriculum, such as engineering.

This chapter describes the math/science workshops* at the University of California, Berkeley, which since 1978 have served a majority of the Black and Hispanic freshmen enrolled in Berkeley’s introductory mathematics and science courses. Instead of providing remediation, the workshops constitute a novel honors program that promotes academic excellence and fosters leadership skills. The program has dramatically affected not only the performance of its participants in their beginning courses, but also their persistence in the university (see the Appendix at the end of this chapter for selected data).

The main principles that distinguish the workshops from other academic support activities will be outlined; the methods that have proven effective will be presented in some detail. It is important, however, to keep in mind that the program has evolved through trial and error. It has been revised, refined, and re-evaluated throughout its existence in order to tailor it to the individual requirements of minority students at Berkeley. Therefore, what follows is offered with the understanding that any program that institutes similar workshops must, of necessity, undergo similar experimentation and adaptation to suit its own needs and to make the best use of its resources.

OBJECTIVES

The workshop program is designed to create an academic environment in which minority students can excel at college-level mathematics and mathematics-intensive science courses. Toward this end, it has four main objectives:

1. To build a community of minority freshmen that focuses on achieving academic excellence and that becomes a source of peer support;
2. To provide extensive, year-long supplementary instruction for minority students;
3. To orient minority students to the university and to assist their adjustment; where necessary, to advocate their collective and individual interests;
4. To monitor the students’ academic progress and to furnish ongoing academic advising.

These objectives, of course, are interrelated. Workshop leaders keep formal instruction to a minimum, while enabling students as much as possible to learn for themselves and from one another. Consequently, the sense of community and the resulting peer support that minority students establish in a workshop are fundamental to the process by which they achieve academic excellence.

Because workshop leaders meet with their workshops several times each week, they can easily monitor their students’ academic progress and adjustment. Thus, they can offer timely advice on the spot. Unlike university counselors or advisors who typically see students only when they are in trouble, the workshop leader can address problems in the making—such as housing arrangements gone awry or delayed financial aid—before they become crises. When a student shows early signs of needing further tutoring, the leader can make the appropriate referrals. The program staff cultivates good working relationships with EOP, affirmative action, and tutoring center staff members, as well as with interested faculty members, so as to coordinate the assistance available to workshop participants.

THE RATIONALE FOR WORKSHOPS

The idea behind a workshop program is that, under the guidance of a skilled teacher, minority students can learn mathematics and science more efficiently by working together with their peers. Many freshmen

* The Math/Science Workshop Program is a project of the Special Scholarship committee of the Berkeley division of the University of California Academic Senate. For further information about this project you may contact Ur. Treisman, Associate Director, Professional Development Program, 230B Stephens Hall, U.C. Berkeley, CA 94720. Telephone: (415) 642-5881.
unhesitatingly embrace the notion of working together, forming groups outside of class to do homework problems and to prepare for examinations. Students in such groups commonly pool their knowledge and their know-how, encourage one another, and find some time in a hectic schedule to relax socially together.

Minority students, however, tend to be isolated from their classmates. In many cases they have achieved academic distinction in high school by keeping apart from less ambitious students and putting in long hours of solitary study. Although the self-reliance that has propelled them to college remains an invaluable attribute, in a workshop they can gain the benefits, both academic and social, of group study. Discussing the course material can uncover misconceptions about how well they have been understanding the lectures and textbook. Comparing approaches to solving problems exposes them to a variety of methods. Assessing their preparation vis-à-vis that of their peers helps them to develop realistic attitudes about what they must do outside of class in order to succeed. Working together in a friendly, cooperative setting can offset the pressures of highly competitive classes.

Despite the benefits that can be enumerated to prospective participants, a workshop program will not appeal to entering minority students if they perceive it to be essentially remediation under a new name. Even though they may be coming from high schools where they were insufficiently prepared, they usually have outstanding high school records. Thus, they are not inclined to view themselves at the outset as needing remedial assistance. Those students who are less prepared than classmates from superior high schools, however, may not recognize their deficiencies until midterm or later, when they perform poorly on an examination. At this point, it is unlikely that tutoring will enable them to make up lost ground.

A workshop program should be considered an honors program both in name and in fact. The students take up difficult problems, and the pace is challenging. When students discover that they need further help or that they are weak in a particular area, they receive individual assistance. This assistance can take place either privately during a workshop session or in supplementary sessions with a tutor. Yet, because in the workshop the student continues to tackle new material at a high level, remedial effort becomes part of a larger goal—not merely to catch up but to excel.

Traditional remedial programs differ from a workshop program in this important respect: remediation merely aims at bringing a student up to a passing grade in order to get through the course. Specialists in learning difficulties identify and address the weaknesses that will prevent a student from earning a "C." But a student who barely scraps by in first-semester calculus will go to the next course in danger of falling behind again. The operating assumption of a workshop staff is that the students are capable of honors grades—that they can earn "A's." Remediation can be accomplished along the way to successful performance. Rather than emphasizing weaknesses, the program builds on the strengths of its students: motivation, determination, high personal standards. But instead of pursuing their goals in isolation, the students can join forces, thereby helping themselves and each other.

HOW A WORKSHOP OPERATES

A visitor to a workshop session might initially mistake it for a noisy study hall where the instructor has temporarily stepped out. Most of the students are talking, some are getting up to look at other students' papers, some appear to be joking. A few are quietly working on a problem, ignoring the buzz around them.

But after a few moments the impression of disorder evaporates, and the purposeful nature of the activities becomes clear: the leader is circulating unobtrusively, alert to the dynamics of the clusters of students, all of whom are wrestling with a set of problems on a worksheet. From time to time, the leader will sit down, a short distance behind a group of three or four students. If they have no questions, the leader will observe them without commenting. If they have hit a snag, they may ask for a hint as to how to proceed. Moving from group to group, the leader can constantly monitor the students' progress by watching them in the act of solving problems.

A workshop is not just a place where students gather to study outside of class, nor is it an extra recitation section. Usually the leader will instruct the workshop as a whole only during the last twenty minutes of a two-hour session. During the rest of the time, both the students and the leaders have carefully defined responsibilities for making the period of group study productive.

The Students' Responsibilities

The students are responsible for attending regularly. Sporadic attendance not only would diminish the absent student's opportunity to profit from the workshop, but would also undermine the sense of community that the workshop is intended to encourage. A student must call in to explain the reason for missing a session—illness or perhaps a test in another subject the next day. Although the best students may correctly decide that they do not need to come to every session, two consecutive absences often indicates that a student is not on top of the material and is having difficulty with the course.

Before attending the workshop, the students are responsible for doing all the classwork they are able to do on their own. They should come with as much of the homework completed as possible, having read the assignments and reviewed their lecture notes. They can then participate actively in the workshop. If the student comes to the workshop unprepared, it will be
apparent to the workshop leaders.

During each session, some time can be spent on homework problems, and students can request help with the problems they have not completed. Some of the worksheet tasks will be devoted to breaking down homework problems into manageable steps. But the main purpose of the session is not to do course assignments but to reinforce concepts and skills by doing extra work: problems xeroxed from old tests that treat material now being studied, extensions of homework problems that introduce more advanced topics, problems that will lead into the planned instruction for the last portion of the workshop.

The worksheet is the vehicle for involving the students in groups, but there are no fixed rules about how they must proceed. Some will elect to start off alone, joining others after a period of quiet thought or exploratory attempts at a problem of their choice. At any one moment, different groups will be discussing and working out different parts of the worksheet. The composition of the groups may vary at each meeting, and within a session individuals may spend part of the time in one group and then leave to work with another. As they work together, they alternate giving and receiving assistance. Since the problems are designed to be tough, there is no stigma attached to asking a fellow participant for help, and the students quickly appreciate the chance to compare papers to see the methods and approaches others take to the same problem.

The students are responsible for critiquing one another’s work. To do this, they must become more articulate critics than they are usually accustomed to being; they must refuse to accept an answer unless they can see why it is correct. It is not unusual at the first sessions for the students in a group to be impressed by one member who is particularly assertive, even when wrong. Nor is it typical for several students to insist they have the same answer when they are patently at odds. The desire to agree—and to be agreeable—overrides their critical perceptions. But as they continue, they learn to question and to demand explanations. Then they are more ready as well to defend their own ideas.

The Leader’s Responsibilities

Leaders must, of course, be thoroughly trained in the discipline that is the subject of a workshop. They do not, however, use workshop time to duplicate the sort of instruction the students receive from a teaching assistant in a recitation section. Instead of standing at a blackboard and demonstrating their expertise or taking the students step-by-step through examples, they design the tasks the students will work on, and they assist the students in learning how to work together.

Deciding what the students should concentrate on in each workshop is one of the leader’s most important responsibilities. Observation of the students as they actually do the problems reveals that they need to expend more effort in certain areas without unnecessarily repeating already familiar procedures. The leader also frequently requests informal feedback from these students outside of the workshop.

The strategy in planning the worksheet is to keep the students challenged with tasks that they recognize as useful for their better understanding of the course material. The problems are set up so that students will get stuck or come to incorrect conclusions. Then the leader can offer obviously useful instruction. There is almost always a problem so difficult that no one can tackle it, and rarely is a worksheet finished. But at the end, when the students have all worked on some of the same problems, the leader will discuss them, often pointing out tricks that the textbook does not cover.

At a weekly curriculum meeting, all the leaders of workshops in one subject construct the worksheets for the next sessions. They analyze the course curriculum and the demands it makes on the students’ mathematics background and problem-solving skills. They design the worksheet accordingly. Along with problems linked to the upcoming weekly homework assignment, worksheets typically include examples of confusing test questions, as well as problems that might look easy but that are quite difficult and vice versa. Putting together the worksheets for each week usually takes three to four hours. (See sample worksheets in Appendix B to this chapter.)

At every workshop session, before the students turn to the worksheet, the leader notes on the board the names of any who are missing. Late students erase their names when they arrive. Students volunteer information if they know why someone is absent. The careful record of attendance allows the leader to follow up quickly in the case of the student who has missed more than once. The leader may even go to the student’s residence to find out why a student has not come and to offer help with any obstacles preventing attendance.

The first workshop meeting poses an especially delicate situation. Unless the students see at once that the workshop can be useful to them, they will not wish to commit to it the extra hours it demands. Yet they can fully appreciate the benefits only gradually. Therefore, the first time—and perhaps the next few times—the leader will operate more traditionally, giving the students some bits of instruction that they can apply directly. The worksheet for the first day is designed to convince the students that they have to work hard but that the effort will result in their doing well academically. They are presented with problems related to the first homework assignment that initially they cannot solve, but that they can do easily once shown how.

While the students are becoming accustomed to studying together, the leader will employ various tactics to sharpen their critical responses. One such exercise involves having two or three students redo another student’s written problem solution. They are asked to
make small changes in language that they think will improve a test score. This task aims at showing them how to accumulate partial credit for style, even if they have a wrong answer.

At first, the students often discover that they are having a hard time communicating with one another. The leader may ask one member of a group to restate a concept more precisely or to explain in more detail the steps taken to arrive at an answer. By making their ideas clearer to the group, students clarify and reinforce their own understanding and find out what they have only superficially grasped. Paying close attention to how they express themselves also helps to improve how they will write on examinations.

In the course of monitoring the groups, the leader may react to a number of situations by taking a student aside for a brief period of intense private instruction. If, for example, someone seems reluctant or is otherwise unable to begin, the leader can suggest practical techniques for getting started: writing anything that comes to mind, drawing a picture, listing facts, looking up definitions and similar problems. Individuals who are unhappy may be experiencing difficulty in adjusting to the university. In some cases, a student may be referred to a professional counselor, especially for example, if alcohol or recreational drugs are impinging on academic performance. The leader also encourages students who are not speaking up in a group to share their work and, thus, draws them out.

In the last portion of the workshop, after the students have struggled with the worksheet, the leader will go over some of the problems they have all attempted. They can then evaluate alternative approaches and isolate key techniques. At this time, they can raise questions about homework assignments and other concerns. To gain perspective on their efforts, they are asked to consider how the material they have been learning fits into the course as a whole.

Before examinations, the leaders will not only review sample test questions on the worksheets but also make suggestions to help the students organize their study outside the workshop: to figure out what should be memorized and what should not; to copy their lecture notes and fill in gaps; to reread the textbook; to check the textbook for clues to likely test questions. In order to develop their ability to allocate their private study time most efficiently, the students practice predicting what the test will look like—which questions will appear and how much certain questions might count in the grading.

The leader can also take responsibility for scheduling additional activities outside the workshop. Extra sessions can be spent on refining the reading and writing skills that are vital for academic success in mathematics and science: reading the technical language in which mathematical tests are written; writing homework, tests, and laboratory assignments in standard mathematical language and form.

As part of their continuing orientation, all the students in a workshop may attend cultural events on campus. From time to time, the leaders may arrange for them to sit in on a lecture in a subject far removed from their regular classes, such as art history of the Renaissance or ancient Turkish poetry. These breaks from the normal routine supply occasions for socializing as well as introductions to the varied resources of their environment.

SETTING UP AND RUNNING A WORKSHOP PROGRAM

A workshop program can be started with little in the way of special equipment—a few rooms containing movable desks or long tables and a board.

But some matters involve extensive planning: recruiting and selecting students; gaining workshop students’ admission in a block to their classes and scheduling students into the appropriate workshops; attending to staff training and program development. Once a program has achieved a reputation for its participants’ academic success, numbers can multiply rapidly, accompanied by increasingly complex administrative responsibilities.

Recruiting and Selecting Students

Students should be recruited during the summer before they enter college. Where there is an MEP program in operation, recruiting minority engineering students will be a normal component of the program. Where no MEP program exists, any college preparatory program for minority high school students, such as MESA, can identify candidates. In the absence of such sources, the names of target students can be compiled from admissions information.

A letter and possibly a follow-up call invite students to an orientation meeting. The orientation consists of an introduction to the workshop program explaining how the workshops function, followed by a pre-calculus test to assess the level of the mathematics course a student should take first. Together with the results of the test, the student’s high school record, SAT scores, and a personal interview provide information to be used in selecting students.

By mid-summer, students are interviewed by a workshop staff member. The interview serves three purposes: (1) to reinforce the main points addressed at the orientation, namely, that this is a challenging honors program and that it will demand considerable time and effort; (2) to determine whether a student should be accepted into the program, based on high school performance and test scores, and also on a student’s commitment; (3) to advise the student, whether or not the student is accepted, about appropriate first-term courses. The interview can also explore potential external problems that may hamper a student academically. At this stage, as well as throughout the year, it is important that the workshop staff be well-
informed about college and departmental requirements for various majors and other matters that can affect the advice given to students.

If, in addition to local students, potential participants are not in the area during the summer, the workshop staff may hold interviews at other locations. If this is not feasible, some last-minute interviews can take place when those students arrive on campus.

At the interview, students are strongly advised to take the minimum number of units during their first term. Typically, this strategy goes against the students' more ambitious inclinations. Especially if they are worried about financial aid, they will want to sign up for more rather than fewer credits. It is hard for them to estimate beforehand how much time they will actually be spending on studying outside class beyond the time spent in workshop.

The interviewers can point out, however, that once they have acquired a solid foundation, they can undertake a heavier credit load in following terms. Until then, should they have any free time, they can look into extracurricular activities—such as student organizations in their area of career interest—that can expand their awareness of future opportunities.

Placing Students in Workshops

One of the most pressing administrative tasks is to match students with courses and workshops. During the pre-enrollment period, the number of workshops to be offered in each course must be set, and the time each workshop will be held must be coordinated with the scheduled times for lectures, sections, and labs in a particular course. In order to construct the worksheets, prepare students for examinations, and otherwise tailor the instruction to course assignments, all the students in a workshop must be attending the same lecture so that they will have the same homework assignments and prepare for the same tests. If clusters of workshop students are in the same sections as well, they can give one another further support. Special arrangements to place workshop students into lectures and sections can best be accomplished when the workshop staff has established cooperative relationships with faculty members who endorse the program's objectives.

Enrolling students in courses and in workshops present similar complications in the second semester. Furthermore, whether to keep students together with the same leader and whether to admit new participants, and if so how to fit them in, are policy matters that will arise. Students who do not pass the fall course can pose a delicate problem. There is also the need to continue advising students about their options and about requirements.

Staff Training and Development

Given the untraditional role of the workshop leader, finding candidates who can combine tact and ingenuity in teaching with informed, sensitive counseling is of paramount importance. Graduate students often have too many commitments to their own work. A graduate program for mathematics and science teachers can be a better source. Also, talented undergraduates, especially if they have gone through the workshops, can become valuable assistants and leaders. A staff made up of faculty members, graduate students in education, and undergraduates can provide a mixture of new and experienced leaders in balanced proportions.

The initiation of new leaders should begin prior to the selection of students for the fall semester. A three-to-four day training session can introduce the program's principles and demonstrate teaching methods. New leaders can attend the general orientation and sit in with experienced staff members on the individual interviews for prospective students.

During the year, various activities can assist both new and experienced leaders in evaluating the quality of instruction. In addition to the weekly curriculum meeting to design the worksheets, leaders teaching the same subject should meet to discuss how best to present material, drawing on different staff members' expertise in particular areas of the curriculum. Leaders also can meet periodically to assess their performance. An individual can describe a difficult situation that has arisen—an instructional matter or perhaps an issue concerning the appropriate response to a particular student's case—for the leaders as a group to analyze.

In order to assess their strengths and weaknesses, two leaders can exchange workshops for a day. The visiting leader solicits from the students criticism and suggestions for the regular leader to consider about improving the instruction. The visiting leader can generate thoughtful analysis by asking each student to compile a list of what occurs in the workshop that is useful to the individual and to the participants as a whole and another list of what is least useful from these perspectives. The students can write their comments separately and then discuss them in groups, and the visiting leader can help them to focus on the most significant issues.

Students can have more opportunities for making suggestions and offering criticism if they are included on committees with leaders and staff members. The process of involving students at this level not only encourages constructive feedback from those directly affected, it can become an extension of the group problem-solving techniques that they are taught in the workshops. For the leaders there is the advantage of receiving student comments frequently throughout the year.
CONCLUSION

A successful workshop program is one in which the participants achieve their academic goals. The following charts document the remarkable success of the Math 1A (first semester calculus) workshops at Berkeley. Similar results could be shown for Math 1B (second semester calculus). The ultimate objective of any such program, however, extends beyond the freshman year of college. It is to develop minority students into independent learners who will go on to complete the major of their choice and begin graduate study or a career in the sciences, engineering, business administration, or other technical fields.

The record of the program’s graduates at Berkeley compared to that of minority students who have not taken part strongly suggests that the workshops do contribute to students’ persistence in the sciences and engineering. Having been through the workshop program as a freshman, however, does not guarantee that a student will sail smoothly through the next several years. Placing students with mentors—upper division students or even faculty members—can be helpful. There is room for further research and experimentation into the ways in which minority students can best be served during and after the first year of college. But results to date indicate that a workshop program can start them on the path to academic excellence.

APPENDIX
Selected Data

Evaluation has been a regular, ongoing activity within the MSP project since its inception in 1978. Results to date suggest that the project has been highly successful in meeting its goals. Despite large yearly increases in the numbers of both students served and subjects offered, MSP students have consistently outperformed their non-MSP minority counterparts in each of the project’s target courses. Moreover, in many of these courses, the average grade of MSP students has been equal to or higher than that of their non-minority classmates.

Tables I and II show, respectively, the performance of Black and Chicano freshmen in Berkeley’s Math 1A, and compare MSP and non-MSP students. This begins a rigorous two-year calculus sequence that plays a pivotal role for science and engineering students; it is a prerequisite for all engineering and physical science majors, and it is the preferred calculus sequence for business administration and biological science majors. Yet, as the charts show, in the five years prior to the creation of MSP, the average Math 1A grade earned by Black and Chicano students was between D+ and C−; the class average for all students during this time was between C+ and B−, a full grade higher. Equally important, but not reflected in the charts, is the fact that more than one-fourth of the minority students who attempted Math 1A during these years dropped the course before completing it, and therefore did not earn a grade. In contrast, with only two exceptions, every minority MSP student who has enrolled in Math 1A during the last three years has completed the course.
TABLE I
Performance of Black Freshmen in Math 1A, by Year of Entry into the University

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<tr>
<td>MSP Students</td>
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<td>25</td>
<td>28</td>
<td>39</td>
<td>40</td>
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<tr>
<td>Non-MSP Students</td>
<td>16</td>
<td>21</td>
<td>28</td>
<td>30</td>
<td>39</td>
<td>30</td>
<td>30</td>
<td>39</td>
<td>15</td>
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</tbody>
</table>

\(\triangle = \text{MSP Participants}\)

\(\circ = \text{Non-MSP Students}\)

Average Math SAT score of Black MSP students in Math 1A during 1977-81 was 505.

Average Math SAT score of Black Non-MSP students in Math 1A during 1973-81 was 499.
TABLE II

Performance of Chicano Freshmen in Math 1A, by Year of Entry into the University

<table>
<thead>
<tr>
<th>Average Grade Points</th>
<th>3.0</th>
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<tr>
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<td>1.2</td>
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<td>Minimum passing grade</td>
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YEAR OF ENTRY | Fall 1973 | Fall 1974 | Fall 1975 | Fall 1976 | Fall 1977 | Fall 1978 | Fall 1979 | Fall 1980 | Fall 1981 |
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<tr>
<td>MSP Students</td>
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<td></td>
<td>10</td>
<td>7</td>
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<td>Non-MSP Students</td>
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<td>15</td>
<td>15</td>
<td>26</td>
<td>20</td>
<td>25</td>
<td>25</td>
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</tbody>
</table>

\[\Delta = \text{MSP Participants}\]

\[\bigcirc = \text{Non-MSP Students}\]
Appendix B

A Sample Math Worksheet
for Second Semester Calculus

MATH 1B  W.S. #32

I (a) Which is larger: \( \int_a^b \sqrt{1 + (f(x))^2} \, dx \) or \( \int_a^b f(x) \, dx \)?

(b) Show that \( \frac{1}{2} \leq \int_0^1 \sqrt{1 + x^2} \, dx \leq \frac{1}{\sqrt{2}} \)

(c) Show that \( \frac{\pi}{2} \leq \int_0^{\frac{\pi}{2}} \sqrt{1 + \cos^2 x} \, dx \leq \frac{\pi}{\sqrt{2}} \)

(d) Evaluate \( \int_0^{2\pi} \sqrt{1 - \cos^2 x} \, dx \) (Hint: the answer is not 0)

II. How many errors can you find? Which can be fixed?

Problem: Find the length of the graph of \( f(x) = \ln \cos x, 0 \leq x \leq \frac{\pi}{2} \)

Solution: \( f(x) = \ln \cos x \Rightarrow f'(x) = \tan x \)

\[ L = \int_0^{2\pi} \sqrt{1 + \tan^2 x} \, dx = \int_0^{2\pi} \sec^2 x \, dx = \left[ \sec x \right]_0^{2\pi} = \frac{\pi}{2} \ln(1) - \frac{1}{2} \ln(1) = 0 \]

Use \( u = \sin x \), & use partial fractions

III (a) Let \( f(x) = mx + b \) \( m > 0 \). Using "similar \( \Delta \)'s" show that the length of the graph for \( a \leq x \leq b \) is \( \sqrt{1 + m^2} \, (b - a) \). Check using arc length formula \( (*) \quad L = \int_a^b \sqrt{1 + f'(x)^2} \, dx \)

(b) Use part (a) to explain why the arc length formula \( (*) \) is plausible.
IV. (a) Express the length of the graph of \( y = \sin x \)
\( 0 \leq x \leq \frac{\pi}{2} \) as an integral. [Do not evaluate]

(b) Argue "geometrically" that
\[
\int_0^{\frac{\pi}{2}} \sqrt{1 + \cos^2 x} \, dx > \sqrt{1 + \frac{\pi^2}{4}} \approx 1.86 \quad \text{(compare with problem Ic.)}
\]

(c) Show that for \( n \geq 1 \)
\[
\int_0^1 \sqrt{1 + (nx^{\frac{n-1}{2}})^2} \, dx \geq \sqrt{2}
\]

(d) Evaluate \( \lim_{n \to \infty} \int_0^1 \sqrt{1 + (nx^{\frac{n-1}{2}})^2} \, dx \) (A picture is worth 2\(^3 \cdot 5^3\) words)

V. (a) Show that \( ab = \frac{1}{4} \Rightarrow 1 + (a-b)^2 = (a+b)^2 \)

(b) Find the length of the graphs of the following functions on the domains indicated.

(i) \( y = \frac{1}{6} x^3 + \frac{1}{2x} \) on \([1,3]\)

(ii) \( y = \frac{x^4}{8} + \frac{1}{4x^2} \) on \([1,3]\)

(iii) \( y = \sqrt{1 - x^2} \) on \([0,1]\)

(iv) Do problem (iii) by another method.
1. 250 ml of 1.00 M NH₃ solution is titrated with 2.00 M hydrochloric acid. Fill in the Table below:

<table>
<thead>
<tr>
<th>mls H⁺ added</th>
<th>pH</th>
<th>[NH₃]</th>
<th>[NH₄⁺]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) 66.5</td>
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<tr>
<td>c) 99.75</td>
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<td></td>
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<tr>
<td>d) 125.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>e) 250.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw a titration curve for the titration and indicate the buffer region using ( ).

If .01 moles of additional HCl were added to the solution in (c), what would the new pH be?

2. A solution is saturated with 0.300 M H₂S gas, (meaning it is 0.300 M H₂S(aq))
What is the solubility of Bi₂S₃ in this solution?

\[ K_a \text{H}_2\text{S} = 9.1 \times 10^{-8} \quad K_{sp} \text{Bi}_2\text{S}_3 = 1.6 \times 10^{-72} \]